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ON MATHEMATICAL TERMINOLOGY: CULTURE CROSSING IN NINETEENTH-CENTURY CHINA

INTRODUCTORY REMARKS: ON MATHEMATICAL SYMBOLISM

The history of mathematical terminologies in nineteenth-century China is a multi-layered issue. Their days of popularity and eventual decline are bound up with the complexities of the disputes between the Qian-Jia School who were concerned with textual criticism and the revival of ancient native Chinese mathematical texts, and those scholars that were interested in mathematical studies *per se*[1] (and which we will discuss in a more detailed study elsewhere).

The purpose of the following article is to portray the 'introduction' of Western mathematical notations into the Chinese consciousness of the late nineteenth century as a major signifying event, both (i) in its own right within the context of the translation of mathematical writings from the West; and (ii) with respect to the way in which the traditional mathematical symbolic system was interpreted by Qing commentators familiar with Western symbolical algebra. Although one might assume to find parallel movements in the commentatorial and translatory practices of one and the same person, nineteenth-century mathematical activities will be shown to be clearly compartmentalized.

In fact, for the period under consideration here, it would be more appropriate to speak of the 'reintroduction' of Western mathematical notations. Already in 1712 the French Jesuit Jean-François Foucquet S. J. (1665–1741) had tried to introduce algebraic symbols in his *Aerrebala xinfa* 阿爾熱八拉新法 (New method of algebra). This treatise, which was written for the Kangxi emperor, introduced symbolical

[1] In particular, the 'three friends discussing astronomy and mathematics' (*Tan tian san you* 談天三友) Wang Lai 汪萊 (1768–1813), Li Rui 李銳 (1763–1820) and Jiao Xun 焦循 (1765–1814). Cf. Horng Wann-Sheng. 1993. "Chinese Mathematics at the Turn of the 19th Century: Jiao Xun, Wang Lai and Li Rui", in: Cheng-Hung Lin and Daiwie Fu (eds.). *Philosophy and Conceptual History of Science in Taiwan*. Dordrecht: Kluwer (*Boston Studies in the Philosophy of Science* 141), pp. 167–208.

Michael Lackner, Iwo Amelung and Joachim Kurtz (eds.). *New Terms for New Ideas. Western Knowledge and Lexical Change in Late Imperial China*. Leiden et al. 2001, pp. 305–326.

algebra for the first time in China[2], using Descartes' notation[3]. However, because of the Emperor's negative attitude towards the 'new' notation, the book exerted no influence on Chinese mathematics at the time.[4]

1. Symbols in translated contexts

In the following analysis of nineteenth-century translations, we will limit our case-study to the (re-)introduction of algebraic notation and, more specifically, to the works of the two earliest mathematical translators Li Shanlan 李善蘭 (1811–1882) and Hua Hengfang 華蘅芳 (1833–1902).[5]

During his eight years in Shanghai, from 1852 to 1860, Li Shanlan translated an impressive number of scientific treatises. Among these

[2] Cf. Catherine Jami. 1988. "Western Influence and Chinese Tradition in an Eighteenth-Century Chinese Mathematical Work", *Historia Mathematica* 15, pp. 311–31.

[3] In early seventeenth-century Europe, two structurally different but equally coherent formulations of algebraic notation, developed by François Viète and René Descartes, co-existed. Cf. Erhard Scholz (ed.). 1990. *Geschichte der Algebra: eine Einführung*. Mannheim: BI-Wissenschaftsverlag (*Lehrbücher und Monographien zur Didaktik der Mathematik* 16).

[4] Horng Wann-Sheng. 1991. *Li Shanlan: The Impact of Western Mathematics in China during the Late 19th Century*. Ph.D. diss., City University of New York, pp. 16–7, describes Fouquet's failed attempt as follows: "Unfortunately, the Emperor did not understand the meaning of the multiplication of symbols: 'Jia multiplied by Jia, Yi multiplied by Yi, do not give any number, and one does not know the value of the result', was the comment in a note duly transmitted to Foucquet: 'It seems to me that [Foucquet's] algebra is very plain and insufficient. In a word, it is laughable.'" Quoted in Jami 1988.

[5] For a more extensive list of Chinese adaptations of European mathematical works, see Jean-Claude Martzloff. 1997. *A History of Chinese Mathematics*. Translated by Stephen S. Wilson. Berlin: Springer, Appendix I. References to other mathematical translations by Hua Hengfang can be found in Li Wenlin. 1996. "The Chinese Indigenous Tradition of Mathematics and the Conceptual Foundation to Adopt Modern Mathematics in the 19th Century," in: Wu Wenjun 吳文俊 et al. (eds.). *Zhongguo shuxueshi lunwenji* 中國數學史論文集 (Collected essays on the history of Chinese mathematics). Jinan: Shandong jiaoyu chubanshe, vol. 4, pp. 146–56; and Wang Yusheng 王渝生 . 1992. "Hua Hengfang" 華蘅芳 , in: Du Shiran 杜石然 et al. (eds.). *Zhongguo gudai kexuejia zhuanji* 中國古代科學家傳記 (Biographies of scientists in ancient China). Beijing: Kexue chubanshe, vol. 2, pp. 1245–52. Qian Baocong 錢寶琮 . 1992. *Zhongguo shuxueshi* 中國數學史 (A history of Chinese mathematics). 3rd edition. Beijing: Kexue chubanshe, pp. 335–7, gives a general survey of Hua Hengfang's own writings, which were profoundly influenced by Western mathematics.

were two mathematical books that were the first works to (re-)introduce symbolical algebra in China.[6] The *Dai weiji shiji* 代微積拾級 (Series of differential and integral calculus) was translated by Li Shanlan in collaboration with the British missionary Alexander Wylie (1815–1887) in 1859.[7] The book was based on Elias Loomis' (1811–1889) *Elements of Analytical Geometry, and of Differential and Integral Calculus*.[8] Following the "Preface"[9] to the *Dai weiji shiji*, Li and Wylie appended a table of 'Chinese' correspondences for the original symbols and the English expressions as well as some explanations.[10] The table and the explanations equally applied to a second work on algebra that Li and Wylie co-translated in the same year. The *Daishuxue* 代數學[11] (Algebra) was a rendering of Augustus De Morgan's (1806–1871)[12] *The Elements of Algebra Preliminary to the Differen-*

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[6] At about the same time, the first work introducing Western mathematical symbols, transmitted via books brought in by merchants from the Netherlands, appeared in Japan. It was written in 1858 by Yanagawa Shunzō. See Yanagawa Shunzō 柳河春三. 1976 [1858]. *Yōsan yōhō* 洋算用法 (How to use Western mathematics). Reprinted in: *Edo kagaku koten sōsho* 江戸科學古典叢書 (Anthology of classical works on science in the Edo period). Tokyo: Kowa shuppan, vol. 20, pp. 127–392.

[7] Li Shanlan and Alexander Wylie (trs.). 1859a. *Dai weiji shiji* 代微積拾級 (Series of integral and differential calculus). Shanghai: Mohai shuguan.

[8] Elias Loomis. 1851. *Elements of Analytical Geometry, and of Differential and Integral Calculus*. New York: Harper & Brothers. Mei Rongzhao 梅榮照. 1960. "Woguo di yiben weijifenxue de yiben—*Dai weiji shiji* chuban yibai zhounian" 我國弟一本微積分學的譯本《代微積拾級》出版一百周年 (The 100th anniversary of the publication of *Dai weiji shiji*, the first translated book on differential and integral calculus in China), *Kexueshi jikan* 3, pp. 59–64, gives a general description of the mathematical contents of this book.

[9] See Appendix 1.

[10] Cf. the "General rules" (*fanli* 凡例) appended to this table: "All the notations found in the present book have not been part of ancient mathematical books. Therefore, let us explain them in detail. '⊥' stands for positive, addition. '⊤' stands for negative, subtraction. We subtract right from left. '÷' stands for division. Right divides left. We can also draw a '—'. The divisor is above, the dividend below."

[11] In any Western algebraic expression, such as the identity $(x\text{-}y)(x+y) = x^{2+}y^{2}$, the letters x and y are said to be variables, by which is meant that they denote arbitrary individual numbers in the sense that any particular number may be substituted for x and y. Hence the Chinese term *daishu* 代數, lit. 'substitute number'.

[12] Most historians have acknowledged De Morgan as a creator of symbolical algebra and therefore an early formalist. Helena M. Pycior. 1983. "Augustus De Morgan's Algebraic Work: The Three Stages", *Isis* 74 (272), pp. 211–26, gives a detailed analysis of De Morgan's changing attitude towards algebra and symbolical algebra. She points out that in 1835 "De Morgan embraced an extremely modern, abstract approach to algebra in particular and mathematics in general". Pycior 1983, p. 211.

tial Calculus.[13] The *Daishuxue* was devoted only to "the development of such parts of algebra as are absolutely requisite for the study of the differential calculus, the most important of all its applications".[14]

A particularly interesting example for the syncretistic symbols presented in both translations combines notations from thirteenth-century Chinese algebra and the Western alphabet. For the first 20 letters of the alphabet (a, b, c, etc.[15]) the ten heavenly branches and the twelve earthen stems are used. The four characters for 'objects' (*wu* 物), 'heaven' (*tian* 天), 'earth' (*di* 地) and 'men' (*ren* 人) correspond to the unknowns w, x, y and z in the source text. It is particularly interesting to note that in the 'algebra' of the 'four elements' (*siyuan* 四元)[16] , as developed in a Yuan dynasty treatise,[17] the expressions 'heaven', 'earth', 'men' and 'objects' (*tian, di, ren, wu*) denote the first, second, third and fourth unknown in the algorithmic descriptions of solution procedures. Yet, the translators distorted the classical order from *tian,*

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[13] Li Shanlan and Alexander Wylie (trs.). 1859b. *Daishuxue* 代數學 (Algebra). Shanghai: Mohai shuguan. Orig. Augustus De Morgan. 1835. *The Elements of Algebra Preliminary to the Differential Calculus, and fit for the higher classes of schools in which the principles of arithmetic are taught*. 2nd edition. London: Printed for Taylor and Walton.

[14] De Morgan 1835, "Preface", p. 4.

[15] In the original, these were in general used for known quantities. In the West, François Viéta was the first to introduce algebraic symbols for known quantities. "After that Algebra had in such manner as is before said, been entertained and cultivated in Europe, and had been so carried on as to reach all forts of *Quadratick Equations*, and in good measure to those of *Cubick Equations* also; (as is to be seen in the Authors mentioned in the former Chapter:) *Franciscus Vieta*, (about the Year 1590,) added a great improvement to it, by introducing what we call *Specious Arithmetick*; which gives Marks or Notes, not only to the Quantities *Unknown*, but to the *Known Quantities* also; and exercises all the Operations of Arithmetick in such Notes and Marks as were before exercised in the common Numeral Figures." John Wallis. 1685. *A Treatise of Algebra: both Pictorial and Practical shewing The Original, Progreßs, and Advancement thereof, from time to time; and by what Steps it hath attained to the Heighth at which now it is*. London: John Playford, Chapter XIV "*Of* Francis Vieta, *and his* Specious Arithmetick", p. 64.

[16] The single character *yuan* 元 was chosen as 'symbol of quantity'.

[17] Zhu Shijie's *Siyuan yujian* 四元玉鑑 (Jade mirror of four elements), first published in 1303, is the only extant source. Yet, in the foreword written by the author's friend Zu Yi 祖頤 , we find references to earlier writings that used algebraic methods with two or three unknowns; the first introduction of a fourth element, the 'object element' (*wuyuan* 物元), is ascribed to Zhu Shijie. Cf. Zhu Shijie 朱世傑 and Luo Shilin 羅士琳 (comm.). 1937 [1836]. *Siyuan yujian xicao* 四元玉鑒細草 (The *Jade mirror of four elements* with detailed calculation sketches). 24 *juan*. Shanghai: Shangwu yinshuguan, vol. 1, pp. 5–8.

di, ren, wu to *wu, tian, di, ren*. This seems to have been a deliberate choice, either in order to follow the (unwritten) convention that x, just like *tian,* represented the first unknown, or to obtain a phonetically more coherent sequence with the alphabetical subset *w*, x, y, z. Letters from the Greek alphabet were translated by using 26 out of the 28 lunar mansions (*xiu* 宿).[18]

There was another mode of creating mathematical symbols which consisted in composing and decomposing Chinese characters. For the respective capital letters of the Latin and Greek alphabets, a character-symbol was created by adding the mouth radical to the left of the corresponding character (*xia*, …). For *∫* (integral) and *d* (differential) the leftmost radicals of the respective Chinese characters translating 'integral' (*wei* 微) and 'differential' (*ji* 積) served as symbolic translations.

Following most of the terminology and symbolic transcriptions developed by Li Shanlan, Hua Hengfang took down in Chinese and translated together with John Fryer (1839–1928) another text on algebra in 1873: the *Daishushu* 代數術 (The art of algebra) was based on William Wallace's (1768–1843) article on "Algebra" for the eighth edition of the *Encyclopaedia Britannica*.[19]

The translation is less 'advanced' than Li Shanlan's with regard to the introduction of operational symbols, since for division, the use of '÷' is not adopted. Instead, a proto-grammatical layout for divisor and dividend is borrowed from the way fractions were already displayed in Li and Wylie's translations:

> In general, when quantities (*jihe* 幾何) are fractionated by quantities, we note the value that results from their division. The method is as follows: Draw a line to separate divisor and dividend. What is above the line makes the divisor, what is below the line makes the dividend. Like $\frac{三}{一二}$, which means that 12 is divided by 3, which again means that the result of this division is 4. Or like $\frac{Yi}{Jia}$, which means that if *jia* is divided by *yi*, the result is *yi* parts out of *jia* (*yi fen zhi jia* 乙分之甲). When we

[18] In earlier translations on Euclidean geometry and trigonometry, the heavenly branches, the earthen stems and the lunar mansions were used to designate points in geometrical figures. Cf. Catherine Jami. 1990. *Les méthodes rapides pour la trigonométrie et le rapport précis du cercle (1774)*. Paris: Collège de France (*Mémoires de l'Institut des Hautes Études Chinoises* 32), pp. 100–1.

[19] Hua Hengfang and John Fryer. 1873. *Daishushu* 代數術 (The art of algebra). Shanghai: Jiangnan zhizaoju. Orig. William Wallace. 1861. "Algebra", in: *Encyclopaedia Britannica. Eighth Edition*. Edinburgh: A. and C. Black. Cf. Adrian A. Bennett. 1967. *John Fryer: The Introduction of Western Science and Technology into Nineteenth-Century China*. Cambridge, Mass.: Harvard University Press, p. 84.

> name this kind of expression, we refer to it as an expression for fractional numbers (*fenshushi* 分數式).[20]

Obviously, arranging the divisor above the dividend is inverse to what we can find in the original texts. Nevertheless, if we consider the fact that in the nineteenth century Chinese texts and mathematical formulae were still written in vertical columns, we can indeed speak of a proto-grammatical choice of layout. The arrangement $\frac{三}{一二}$, for example, corresponds precisely to the Chinese formulation in traditional mathematical language, i.e. 'three parts out of twelve' (*san fen zhi shier* 三分之十二). It is thus more 'natural' for a reader to see the divisor above the dividend, with the horizontal line between them symbolizing the syntactical unit 'parts out of' (*fen zhi* 分之).

For the evaluation of the reception of the (re-)introduction of symbolical algebra, let us look at Xie Chonghui's 解崇輝 direct response to Hua and Fryer's *Art of Algebra*. Printed in 1900, the *Daishushu bushi* 代數術補式 (The art of algebra with further expressions)[21] provided complementary explanations on the symbolic equations used by Hua and Fryer. As Xie states in his own foreword:

> Amongst the recently translated books on algebra, the *Art of algebra* is most easy to read. Yet, it still contains simple expressions (*jianshi* 簡式). … I have expanded (*xiang* 詳)[22] those amongst the original expressions, that were simple.

Interestingly enough, it was not at all the terminological set that attracted the attention of the commentator, but the assimilated symbolic notation. Yet, from the translator's comments cited from the above books, it becomes clear that their problem was twofold: they were confronted with the translation of (visual) signs and (linguistic) concepts foreign to the Chinese algorithmic tradition. Why then is it that despite their formal equivalence with symbols for variables and operations, neologisms had a less privileged claim to attention? First, and most simply, changes in terminology occurred constantly and earlier than the introduction of signs for operations or indeterminate

[20] Hua and Fryer 1873, "Explanation of notations" (*Shi hao* 釋號), p. 4a.

[21] Xie Chonghui 解崇輝 . 1900. *Daishushu bushi* 代數術補式 (The art of algebra with further expressions). Shanghai: Shuncheng shuju.

[22] In the "General rules" in Li and Wylie 1859a, we find an example that illustrates the significance of a 'simple expression' (*jianshi* 簡式) and its 'expansion' (*xiangshi* 詳式). It is $(x+y)^2$ and its expansion $x^2+2xy+y^2$, or in 'Chinese' symbols: $(天\perp地)^{=}$ and its expansion $天^{=}\perp{=}\ 天地\perp地^{=}$.

numbers. Secondly, the obscure but undeniable connection of neologisms to much older and more deeply embedded ideas indicates that questions about their introduction could be phrased in more familiar terms. One had to decide on the impact of the tradition or a foreign language when a new expression for a mathematical object entered the lexicon of the written code. Wylie's Chinese preface to the *Algebra*, for example, clearly shows that due to the rediscovery of many ancient texts, nineteenth-century mathematicians were more conscious about the Chinese mathematical tradition. They were thus more inclined than earlier translators to integrate ancient concepts.[23] But when we look at modern Chinese mathematical terminology, it becomes obvious that by the beginning of the twentieth century, there was a clear preference to adopt Japanese expressions, and most of the terms coined in Li's and Hua's translations fell into oblivion (see Appendix, Table 2). In fact, the historical process of transmission was much more complicated than can be described here. Li's translations were re-edited in Japan.[24] The terminology chosen therein became thus known to Japanese scholars, who again reworked and re-assimilated it into their language and logic. A Committee for the Fixation of Translated Terminology (*Sūgaku yakugokai* 数学訳語会) of the Mathematical Society of Japan (*Nihon sūgakukai* 日本数学会) held regular meetings between 1880 and 1884.[25] The results were probably reintroduced into China during the first decade of the twentieth century when China was strongly oriented towards the Japanese model.

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[23] Cf. for example the adoption of the ancient technical terms for 'positive' and 'negative', in contrast to earlier translations: "Symbols used for the 'Borrowing of Roots and Squares' [expression used in seventeenth- and eighteenth-century writings for Western methods to solve algebraic equations numerically] are rather simple and rough. ' ⊥ ' was used as a sign for addition. This is the same as in today's algebra (*daishu* 代數). Its former name was 'many'. Today we change its name to 'positive' (*zheng* 正). '-' was used as a sign for subtraction. Today we use ' ┬ '. Its former name was 'few'. Today we change its name to 'negative' (*fu* 負)." Li Shanlan and Alexander Wylie (trs.). 1872. *Daisūgaku* 代數學 (Algebra). Japanese edition of id. 1859b. Edited by Tsukamato Neikai 冢本明毅 . Tokyo: Izumiya Ichibei et al., p. 2a.

[24] E.g. Li and Wylie 1872. Cf. Numata Jirō 沼田次郎 et al. (comps. and eds.). 1984. *Yōgaku shi jiten* 洋學史事典 (*Dictionary of the History of 'Western Learning'*). Tokyo: Yushodo Press, pp. 414, 424.

[25] See Annick Horiuchi. 1999. "Langues mathématiques de Meiji: à la recherche d'un consensus?", paper presented at the International Colloquium "Traduire, transposer, naturaliser", Paris, 5–6 October 1999.

In the following two sections, we will focus on how the Chinese language, on a conceptual level, fitted the modes of symbolic denomination. Instead of drawing a clear conceptual line between symbolic and linguistic expressions, special attention is paid to their relation in the process of mathematical culture crossing.

1. The lack of a conceptual boundary between 'signs' and 'names'

The operational and denominative nature of Chinese characters since early times has been pointed out on several accounts.[26] Probably, this characteristic of Chinese mathematical language was a preliminary to the fusion between a symbolic and linguistic practice of nineteenth-century mathematics. Besides the above discussed assimilation of division into the syntax of literary Chinese, the most significant statement to support our hypothesis might be the following one by the translators themselves. It parallels 'signs' to indicate whether mathematical terms should be positive and thus added, or negative and thus subtracted, with the ancient way of indicating this by the prefix *zheng* 正 or *fu* 負 added to the names of mathematical objects:

> In general, when we have several algebraic expressions, which do carry the sign '+' (⊥), as well as the sign '-' (⊤), we call them 'numbers with unequal *signs*' (*bu tong hao*[27] *zhi shu* 不同號之數). We also call them 'numbers with different *names*' (*yi ming shu* 異名數).[28]

To support our argument that this conceptual fluency between signs and names is inherent to Chinese mathematical language, let me give another, more recent, example. Zhang Yong's 章用 (1911–1939)[29]

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[26] Cf. John Hoe. 1977. *Les systèmes d'équations polynomes dans le* Siyuan Yujian *(1303)*. Paris: Collège de France (*Mémoires de l'Institut des Hautes Études Chinoises* 6), pp. 37, who argues that "in Chinese, it is not necessary to define particular symbols a, b, c and d …, since the Chinese characters … designate precisely what we have assigned per definition to the letters a, b, c and d. … Furthermore, the monosyllabic character (*cheng* 乘) from the Chinese text, signifying 'multiplied with', fulfils the same function as the multiplication symbol x employed in modern mathematics".

[27] Although, in general, a more appropriate translation for *hao* 號 would be 'appellation', we translate it here with 'signs', since in the preceding phrase it refers to the operational symbols + and -, and in binomial combination (*jihao*) to 'notations' in general.

[28] Hua and Fryer 1873, "Explanation of notations", p. 2b.

[29] According to Jean-Claude Martzloff. 1992. "Li Shanlan (1811–1882) and Chinese Traditional Mathematics", *Mathematical Intelligencer* 14.4, pp. 2–37; 32, Zhang Yong was "[a] young polyglot Chinese mathematician who was born in Aberdeen, Scotland and who had studied mathematics at Göttingen University … [and] took up Chinese traditional mathematics."

recurrent interpretation of the term *duoji* 垛積, lit. 'accumulation of piles', coined by Li Shanlan in one of his own essays[30], shows that replacing a Chinese character with an algebraic operator allows a consistent explanation of Li's terminological set for arithmetical series:

> If one were to explain the character *ji* 積 in *duoji* 垛積 as the character for summation, then it would correspond to the operator $\sum_{i=1}^{n}$.[31]

2. The problem of signification

Semiotically, it was the algebraic *language*, more precisely the opposition that this (Western) language offered between determinate, possibly unknown, but fixed numbers (constants) and the indeterminate non-numerical entities (variables), that was of primary interest. In Chinese positional 'celestial element' (*tianyuan* 天元) algebra, the art of manipulating polynomials or equations was not co-extensive with the idea of denoting a variable.

As mentioned in the introduction, we wish to portray the encounter of these two mathematical cultures as a major signifying event. The reasons for doing so also lie within the Chinese mathematicians' perception of the introduction of Western mathematical notations into the Chinese discourse itself, and their confrontation to the entailed semiotic problems. As their comments and terminological choices testify, the problem of signification was particularly severe in those cases where no functional relation existed between a symbol (or a name) and a corresponding mathematical reality, i.e. in the case of (i) existing symbols for the absence of anything, and in the inverse case of (ii) the absence of names for existing mathematical objects.

(i) The signs for zero and for infinity, for example, are both structured around the notion of 'nothing', an absence of anything, in the sense of a signified non-presence of numerical actualities. Their respective symbols were presented as beiing related to each other:

> 'o' is nothing (*wu* 無), 'oo' is nothing beyond (*wu qiong* 無窮).[32]

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[30] Li Shanlan 李善蘭. 1867. "Duoji bilei" 垛積比類 (Analogical categories of discrete accumulations), in: id. *Zeguxizhai suanxue* 則古昔齋算學 (Mathematics from the *Zeguxi*-Studio). Jinling: Haining Li Shanlan jinling keben, part 4.

[31] Zhang Yong 章用. 1939. "Duoji bilei shuzheng" 垛積比類疏證 (Explanations and proofs for the *Analogical Categories of Discrete Accumulations*), *Kexue* 23.11, pp. 647–63; 648.

[32] Li and Wylie 1859a, "General rules", p. 2b.

(ii) The absence of discursive names (*wu ming* 無名) for yet well defined objects was no new phenomenon. It occurred in other contexts, as in botany,[33] as well as in mathematics. We will come back to this point in the discussion of Dai Xu's 戴煦 interpretation and extension of a thirteenth-century nomenclature for arithmetical series (see below).

2. Symbolic operations in traditional contexts

When turning to the traditional branch of mathematical activity in nineteenth-century China, our understanding of mathematical symbols first needs to be clarified. No such thing as symbolic algebra can be found in these mainly exegetical writings.

Taking the discipline of Chinese mathematics as a culture-dependent intellectual product, in particular as the production of text and linguistic signs, implies that all linguistic or semiotic entities contained in these texts (names and expressions, tabular arrangements of coefficients or drawings[34]) can be analyzed as mathematical symbols; not only as purely conventional symbols (and therefore 'symbolism' not merely as a form of notation), but also as symbols referring to executable operations. In this way, their semantic scope is defined through the relations between the symbols, and not primarily through their relations to a presupposed geometrical or arithmetical reality.

In order to compare the signifying events within the translation of mathematical writings from the West to the events within the interpretations of the traditional mathematical symbolic system, we will analyze in the following two sections the creation of a new or extended terminological set for finite arithmetical series. Elsewhere[35] we have

[33] Cf. Georges Métailié. 1993. "Plantes et noms, plantes sans nom dans le *Zhiwu mingshi tukao*", *Extrême-Orient—Extrême-Occident* 15, pp. 138–48; 142: "Wu Qixun [1789–1847, author of the *Zhiwu mingshi tukao*, 1848], innovates in his own way by limiting his interest not only to plants which are already widely known: he introduces some twelve unknown and unnamed plants, *wuming*, which do not form a group apart but are inserted into those categories, *lei*, to which he affiliates them." [The translation is mine, A.B.]

[34] Michael Lackner. 1992. "Argumentation par diagrammes: une architecture à base de mots. Le *Ximing* depuis Zhang Zai jusqu'au *Yanjitu*", *Extrême-Orient—Extrême-Occident* 14, pp. 131–68, has emphasized the argumentative role of diagrams or 'maps' (*tu*) in the Chinese tradition.

[35] Andrea Bréard. 1999. *Re-Kreation eines mathematischen Konzeptes im chinesischen Diskurs: Reihen vom ersten bis zum 19. Jahrhundert*. Stuttgart: Franz Steiner (*Boethius* 42), ch. 4.

applied the above understanding of a symbol system in our interpretation of Zhu Shijie's treatment of series in his *Siyuan yujian*. Zhu's semiotic approach to 'series', here better described by the expression 'discrete accumulations', was revisited by later mathematicians and historians of mathematics in various contexts, in particular during the late nineteenth century after the rediscovery of his text.

1. Dai Xu's (1805–1860) extension to nameless objects

Looking at Dai Xu's interpretations of Zhu Shijie's treatise[36], we first notice that he keeps the commentary on discrete accumulations distinct from the classical text. He thus takes a global view on Zhu's terminological system. Detached from the specific problem, such commentatorial reading helped inspire him to classify and correlate terminological sets and mathematical objects. Dai Xu, trying to complete Zhu's system, devised an intricate system of mathematical correspondences between different combinations of accumulation types already named in the classic and three new types of terms that do not appear in the original text ('difference between layers' *ceng cha* 層差, 'accumulation per layer' *ceng ji* 層積 and 'total accumulation' *zong ji* 總積).[37]

His terminological extension inevitably led to 'nameless' (*wu ming* 無名) accumulations since, mathematically speaking, it was completely redundant. In Table 1 below, we find three different linguistic expressions for each of the two central accumulations. Theoretically, it might of course have been possible to extend the set of combinatorial elements, and thus rectify the 'nameless', but we assume that the symbolic nature of Dai's nomenclature had already integrated the idea of a non-signifying symbol for variable referents.

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[36] Zhu Shijie 朱世傑 and Dai Xu 戴煦 (comm.). 1844. *Siyuan yujian xicao* 四元玉鑒細草 (The *Jade mirror of four elements* with detailed calculation sketches). (unpublished manuscript). This manuscript was recently rediscovered and is now in the library of the National Tsing Hua University in Taiwan. Cf. Liu Dun 劉鈍. 1995. "Fang Tai suojian shuxue zhenji" 訪台所見數學珍藉 (Precious old books found during a visit to Taiwan), *Ziran kexueshi yanjiu* 16.4, pp. 8–21. I am particularly grateful to Prof. Huang Yilong from National Tsing Hua University for providing a copy of this text.

[37] Readers interested in the detailed mathematical realities of these terminologies may refer to Bréard 1999, chaps. 4.3 and 5.

Table 1: Nameless four-dimensional accumulations

1	1	1	1	1	1
12	11	10	9	8	7
58	46	35	25	16	8
188	130	84	49	24	8
483	295	165	81	32	8
total accumulation of four-angular foggy mountain peak forms	total accumulation of nameless four-dimensional forms	total accumulation of hyper-layers of a four-angular pile	total accumulation of rectangles	accumulation per layer of rectangles	difference between layers of rectangles
	accumulation per layer of four-angular foggy mountain peak forms	accumulation per layer of nameless four-dimensional forms	accumulation per layer of hyper-layers of a four-angular pile	difference between layers of hyper-layers of a four-angular pile	
		difference between layers of four-angular foggy mountain peak forms	difference between layers of nameless four-dimensional forms		

Source: Zhu and Dai 1844

2. Li Shanlan's extension to a system of triangular diagrams

Published in Nanjing in 1867, the *Zeguxizhai suanxue* 則古昔齋算學 (Mathematics from the *Zeguxi*-Studio) contain essays inspired by Western mathematics on the basis of Jesuit works (logarithms, conic sections, infinite series, prime numbers, etc.) as well as independently developed essays on traditional themes.

As we have seen in the case of Dai Xu, the abandonment of the traditional form of commentary by other modes of discourse did not just provide a new format of expression, but influenced the organization of mathematical knowledge and modes of naming. Based on Zhu Shijie's work from the Yuan dynasty, Li Shanlan presents his "Duoji bilei" 垛積比類 (Analogical categories of discrete accumulations) clearly as one of the essays included in his collection that were written within the traditional mathematical framework without employing 'Westernized' notation.[38]

The triangular diagram as given at the beginning of Zhu Shijie's book remains the starting point of Li's system. For the inverse problems (find the height of the pile when the total accumulation is given), Li represents the solution procedures by a second triangular diagram which contains in each horizontal line the coefficients of the corresponding polynomial equations. Since in the original diagram the elements' arrangement was significant in a diagonal line, Qiang Ruxun強汝詢 severely criticized Li Shanlan's emendation of the classical orientation of reading mathematical triangular tables.[39] Moreover, Li's terminological set is no longer specifically related to the elements of a diagonal line. Their progression type had been crucial for the way in which Zhu Shijie named the associated accumulation types. Li Shanlan operates instead with ordinal numbers to designate the sequence of diagonal lines in each diagram.[40] Therefore, he had to define the mathematical relations between different groups of accumulations

[38] Cf. the preface to this essay: "What we describe here, contains tables, has drawings and methods. It is systematically subdivided into different sections and precisely and meticulously expounded in order to bring the knowledge of the art of discrete accumulations closer to those who study mathematics and to raise a new standard besides the *Nine Chapters*." Li Shanlan 1867, p. 1a.

[39] Cf. Qiang Ruxun 強汝詢 . 1918. "Duoji yanshu" 垛積衍術 (Development of procedures for discrete accumulations), in: Liu Chenggan 劉承幹 (ed.). *Qiushuzhai congshu* 求恕齋叢書 . n.p. 4 *juan*.

[40] The only exception are the *san jiao zi cheng*-piles (三角字乘, lit. 'triangles multiplied by themselves') where Li uses the twelve heavenly branches.

explicitly in an explanatory text; the symbolic names he associated with the mathematical objects no longer performed the relational function as it had been the case for Zhu's terminology.

3. Hua Hengfang's application of Western methods

In the "Duoji yanjiao" 垛積演較 (Development of differences for discrete accumulations)[41], Hua Hengfang solves all problems on discrete accumulations from Zhu Shijie's *Jade Mirror* within the framework of his "Jijiaoshu" 積較術 (Procedure for differences of accumulations). This procedure, which he had expounded extensively some ten years earlier, corresponds mathematically to the Western 'method of finite differences' (*youxian chafen fa* 有限差分法) with which he must have been familiar by that time. Yet, just like Li Shanlan, Hua does not adopt any of the algebraic notations introduced in Li's and his own translations. Instead, he remains within and extends the traditional semiotic framework of names and tabular layouts.[42]

3. Japanese influence?

Through the transmission of Zhu Shijie's *Suanxue qimeng* 算學啟蒙 (Introduction to mathematics, 1299) to Korea, where it was presumably printed in 1433 under the reign of King Sejong 世宗 (1418–1450),[43] and further on to Japan at the end of the sixteenth century, the

[41] Hua Hengfang 華蘅芳. 1893. "Duoji yanjiao" 垛積演較 (Development of differences for discrete accumulations), in: id. *Suancao congcun er* 算草叢存二 (Collection of mathematical sketches 2). Liangxi: Liangxi Hua Hengfang keben (Wuchang ed.).

[42] The contents of this work—Hua Hengfang 華蘅芳. 1882. "Jijiaoshu" 積較術 (Procedure for differences of accumulations), in: id. *Xingsuxuan suangao* 行素軒算稿 (Mathematical sketches from the *Xingsuxuan*-Studio). Liangxi: Liangxi Hua Hengfang keben, part 4—in particular its contribution to 'series' and related 'interpolation techniques', is described in Luo Jianjin 羅見今. 1986. "Hua Hengfang de jishu hanshu he hufan gongshi" 華蘅芳的計數函數和互反公式 (Hua Hengfang's counting functions and inversion formulae), in: Wu Wenjun 吳文俊 et al. (eds.). *Zhongguo shuxueshi lunwenji* 中國數學史論文集 (Collected essays on the history of Chinese mathematics). Jinan: Shandong jiaoyu chubanshe, vol. 2, pp. 107–24.

[43] According to the preface of the Korean re-edition by Kim Si-jin 金始振, in: id. 1660. *Sinp'yōn Sanhak kyemong* 新編算學啟蒙 (New edition of the *Introduction to mathematics*). Chōnju. The first critical edition was published by Kyong Son-jing 慶善徵 (1616–?). Cf. Zhu Shijie 朱世傑. 1839. *Suanxue qimeng* 算學啟蒙 (An introduction to mathematics). 3 vols. Reprint of the 1660 Korean edition. This edition is held in Tokyo. Cf. Ho Peng-Yoke. 1971. "Chu Shih-chieh", in: Charles C. Gillispie (ed.). *Dictionary of Scientific Biography*. New York: Scribner's, vol. 3, pp. 265–71; 265.

Chinese algebraic *tianyuan* 天元 method (Jap. *tengenjutsu*) had a significant impact in East Asia, and many commentaries were written about it. In 1690, Takebe Katahiro 建部賢私 (1664–1739) wrote the seven-volume *Sangaki keimō genkai taisei* 算學啟蒙諺解大成 (Complete commentary on the *Introduction to mathematics* in Japanese vernacular), which spread beyond learned circles. Takebe adopted therein a new notation system on paper developed by his teacher Seki Takakazu 關孝和 (?–1708), the 'method of side marks' (*bosho-hō* 傍書法), which transformed the basic significance of the conceptual framework of the *tianyuan* method.[44] At around 1683, Seki himself wrote a *Katsuyō sanpō* 括要算法 (Compendium of mathematical methods, posthumously published in 1712) which again contains tables and problems concerning discrete accumulations.[45] Yet, there is no historical evidence that Zhu Shijie's later book, the *Jade Mirror*, had also been transmitted to Japan. Seki's terminological choices do not support the hypothesis of a possible transmission either.

The validity of the hypothesis that Li Shanlan knew Seki's work is a different matter. There is no historical evidence either, but a comparison of their classification schemes and terminologies for discrete accumulations shows that there might indeed have occurred a transmission from Japan to China (see Appendix, Tables 3 and 4).

Both authors developed diagrams to calculate the sums of finite series. Both begin with power series and arithmetic progressions up to the same order, though in inverse sequence. The sections concerning the solution procedures[46] explain in both writings the procedure up to

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[44] Sato Ken'ichi. 1995. "Re-evaluation of *Tengenjutsu* or *Tianyuanshu*: In the Context of Comparison between China and Japan", *Historia Scientiarum* 5.1, pp. 57–67, interprets this as the assimilation of Chinese counting rods as mathematical symbols: "Original denotation of counting-rods came to have another implication in Japan; the figures of counting-rods became the system of mathematical symbols. They could write down these symbols on paper without performing counting-rods on the board." Sato 1995, p. 65.

[45] His *Daseki sōjutsu* (General procedures for discrete accumulations) has a special section on a method to solve a system of polynomial functions (*ruisai shōsa hō* 累裁招差法) in *juan* 1. Seki Takakazu 關孝和. 1974. "Daseki sōjutsu" 垜積總術 (General procedures for discrete accumulations), in: id. *Seki Takakazu zenshū* 關孝和全集 (Collected works of Seki Takakazu). Edited by Akira Hirayama 平山諦, Kazuo Shimodaira 下平和夫 and Hideo Hirose 谷瀨秀雄. Tokyo: Ōsaka kyōiku tosho.

[46] Seki's and Li's approaches to calculate the sum of power series is described in Shen Kangshen 沈康身. 1987. "Guan Xiaohe yu Li Shanlan de ziranshu mihe gongshi" 關孝和與李善蘭的自然數冪和公式 (Seki Takakazu's and Li Shanlan's formulae for the summation of power series), in: Wu Wenjun 吳文俊 et al. (eds.). *Zhongguo*

a certain series, f, and then ask to deduce the further ones analogically. The possibility of a Japanese influence on Li Shanlan's nomenclature would thus be worth evaluating, although an explanation simply in terms of common origins in a well-established Chinese tradition cannot be excluded.[47]

CONCLUSION

Without over-generalizing the development of mathematical notations and terminologies in the late nineteenth century, I would suggest it useful to take into account the difference between the writings in continuation of the ancient traditions and the translations from Western mathematical works. Li Shanlan's treatment of series, for example, seems to have been conducted within two different conceptual frameworks: (i) finite series (*duoji* 垛積), i.e. within the traditional tools of triangular tables and the *tianyuan*-layout of coefficients for the corresponding algebraic equations as found in Song-Yuan sources; (b) infinite series (*jishu* 級數), i.e. within a syncretistic notation for equations, interspersed with neologistic algorithmic formulations.

In general, the problems attendant in the crossing of two mathematical systems were elucidated in translations and commentaries with a partly extended set of objects, carrying *new* or *no* names. Specifically, I argued that certain crucial changes in the codes of algebraic expressions (the introduction of *variables*), visual depiction (the integration of *operational symbols* in formulae) and algorithmic descriptions (the creation of *neologisms* in discursive passages) occurred as part of the discontinuity in Chinese mathematical culture. Moreover, I

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46 (*cont.*) *shuxueshi lunwenji* 中國數學史論文集 (Collected essays on the history of Chinese mathematics). Jinan: Shandong jiaoyu chubanshe, vol. 3, pp. 81–93.

47 Note the similar situation of resemblances for infinite series, expressing trigonometric functions: "We can only recognize here the striking similarity that exists between the calculation procedures of Matsunaga [Yoshisuke 松永俊仍 (?–1747), author of the *Hōen sankei* 方圓算經 (Classic of square and circle, 1739)] and those of Ming Antu 明安圖 (died 1763). Ming's methods are expounded in a work entitled *Geyuan milü jiefa* 割圓密率捷法 (Quick methods for trigonometry and the precise proportion of the circle, 1774). … This example illustrates again the impossibility to explain the development of mathematics in China and in Japan merely in terms of influences." Annick Horiuchi. 1994. *Les mathématiques japonaises à l'époque d'Edo—Une étude des travaux de Seki Takakazu (?–1708) et de Takebe Katahiro (1664–1739)*. Paris: Librairie Philosophique J. Vrin, p. 345.

argued that the shifts that took place within these codes exhibit parallel patterns of semiotic disruption from names to symbols, and provided evidence for a tendency towards the absence of linguistic signs due to objects 'without names' (*wu ming* 無名). The creation of mathematical terminology in the late nineteenth century thus turned out to be an inhomogenous search for protogrammatical transcriptions and a syncretism between classical and Western concepts with little impact on the modern Chinese lexicon.

APPENDIX

Alexander Wylie's "Preface" to the Dai weiji shiji 代微積拾級 *(Series of differential and integral calculus)*, *Shanghai, July 1859*

The present work, which is a translation of Loomis' Analytical Geometry, and Differential and Integral Calculus, is issued in pursuance of a project formed some time since, as the continuation of a course of mathematics, the first of which, a Compendium of Arithmetic[48], was published by the undersigned in 1854. The next in order is a Treatise on Algebra, which should have preceded this, but in consequence of unavoidable delays in the publication, it will not be issued till some weeks later. A tolerable acquaintance with the last-named treatise, will put the student in a position to understand the work now presented to the public. Although this is the first time that the principles of Algebraic Geometry have been placed before the Chinese (so far as the translator is aware), in their own idiom, yet there is little doubt that this branch of the science will commend itself to native mathematicians, in consideration of its obvious utility; especially when we remember the readiness with which they adopted Euclid's Elements of Geometry, Computation by Logarithms, and other novelties of European introduction. A spirit of inquiry is abroad among the Chinese, and there is a class of students on the empire, by no means small in number, who receive with avidity instruction on scientific matters from the West. Mere superficial essays and popular digests are far from adequate to satisfy such applicants; and yet when anything beyond that is attempted, the want of a common medium of communication at once appears as an insuperable obstacle; and it is evident that how clearly soever we may be enabled to lay results before the native mind, yet until they understand something of the processes by which such results are obtained, thinkers of the above class can scarcely be supposed to appreciate the achievements of modern science, to repose absolute confidence in the results, or to rest satisfied till they are in a position to some extent to verify the statements which are laid before them. It is hoped that the present translation will in some measure supply what is now a desideratum; and the translator, while taking this opportunity to testify to the exceeding care and accuracy displayed in

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[48] A reference to another book written in Chinese by Alexander Wylie: id. 1854. *Shuxue qimeng* 數學啟蒙 (Compendium of arithmetic). Shanghai: Mohai shuguan.

the work of Professor Loomis, considers it is but justice to the native scholar Le Shan-lan, who has assisted in the translation throughout, to state that whatever degree of perfection this version may have attained, is almost entirely due to his efforts and talents.

A list of technical terms used in the works above-mentioned is subjoined.

Table 2: Comparative list of selected mathematical terms

English (1859)	*Li Shanlan (1859)*	*Japanese (1891)*[a]	*Modern Japanese*[b]	*Modern Chinese*
algebraic geometry	*daishu jihe* 代數幾何	*daisū kikagaku* 代數幾何學	*daisū kikagaku* 代數幾何學	*daishu jihexue* 代數幾何學
approximation	*milü* 密率	*kinzan* 近算 *gaitō* 概當	*kinji* 近似	*jinsi* 近似
axiom	*gonglun* 公論	*kōri* 公理	*kōri* 公理	*gongli* 公理
binomial	*erxiangshi* 二項式	*nikōshiki* 二項式	*nikōshiki* 二項式	*erxiangshi* 二項式
binomial theorem	*hemingfa* 合名法	*nikōshiki teiri* 二項式定理	*nikōshiki teiri* 二項式定理	*erxiangshi dingli* 二項式定理
concave convex	*aotu* 凹凸	*ōtotsu* 凹凸	*ōtotsu* 凹凸	*aotu* 凹凸
construct	*zuotu* 作圖	*sakuzu* 作圖	*sakuzu* 作圖	*zuotu* 作圖
continuity	*jianbian* 漸變	*renzoku* 連續	*renzoku* 連續	*lianxu* 連續
converging series	*lian jishu* 歛級數	*shūren kyūsū* 收歛級數	*shūren kyūsū* 收歛級數	*shoushu jishu* 收束級數
coordinates	*zonghengxian* 縱橫線[c]	*zahyō* 坐標	*zahyō* 座標	*zuobiao* 座標
definition	*jieshuo* 界說[d]	*teigi* 定義	*teigi* 定義	*dingyi* 定義
expansion	*xian shi* 詳式	*tenkai* 展開	*kaishiki* 開式	*zhankai* 展開
frustum	*jieyuanzhui* 截圜錐	*setzutōtai* 截頭体 *tai/dai* 台	*tai/dai* 台	*pingjie touti* 平截頭體 (*tai* 台)
general expression	*gongshi* 公式	*hanshiki* 範式 [= formula]	*kōshiki* 公式 [= formula]	*gongshi* 公式 [= formula]

Table 2: Comparative list of selected mathematical terms (cont.)

English (1859)	*Li Shanlan (1859)*	*Japanese (1891)*[a]	*Modern Japanese*[b]	*Modern Chinese*
incommen-surable	*wudengshu* 無等數	*fujin* 不盡	*futsūyakusei* 不通約性	*buketongyuede* 不可通約的 *wugongyueshude* 無公約數的
lemma	*li* 例	*hodai* 補題	*hojoteiri* 補助定理	*buzhu dingli* 補助定理 *yinli* 引理
notation	*ming wei* 命位 *ji fa* 紀法	*kihō* 記法	*kigō* 記號 *ki(sū)hō* 記 (數) 法	*fuhao* 符號
parabola	*paowuxian* 拋物線	*hōbutsusen* 拋物線	*hōbutsu* 拋物 *hōbutsusen* 放物線	*fangwuxian* 放物線 *paowuxian* 拋物線
parallel	*pingxing* 平行	*heikō* 平行	*heikō* 平行	*pingxing* 平行
polynomial	*duoxiangshu* 多項數	*takōshiki* 多項式	*takōshiki* 多項式	*duoxiangshi* 多項式
postulate	*qiu* 求	*kiku* 規矩	*kōjun* 公準	*jiashe* 假設 *gongli* 公理 *gongshe* 公設 *jiben yuanli* 基本原理
proposition	*kuan* 款 [e]	*meidai* 命題 *dai* 題	*meidai* 命題	*mingti* 命題
quantity	*jihe* 幾何	*ryō* 量	*ryō* 量	*(shu) liang* (數) 量
root	*gen* 根	*kon* 根	*kon* 根	*gen* 根
root of equation	*mieshu* 滅數	*hōteishikikon* 方程式根	*kon* 根	*gen* 根
theorem	*shu* 術	*teiri* 定理	*teiri* 定理	*dingli* 定理
transcendental	*yue* 越	*chōetsu* 超越	*chōetsu* 超越	*chaoyue* 超越

Table 2: Comparative list of selected mathematical terms (cont.)

English (1859)	*Li Shanlan (1859)*	*Japanese (1891)*[a]	*Modern Japanese*[b]	*Modern Chinese*
unknown	*weizhi* 未知	*michi* 未知	*michi* 未知	*weizhi* 未知
value	*tongshu* 同數	*chi/atai* 值	*chi/atai* 值	*shuzhi* 數值
variable	*bianshu* 變數	*hensū* 變數	*hensū* 變數	*bianliang* 變量

Notes: (a) Cf. Fujisawa Rikitarō 藤澤利喜太郎 . 1988 [1891]. *Sūgaku yōgo Ei-Wa taiyaku jisho* 數學用語英和對譯字書 (Vocabulary of mathematical terms in English and Japanese). 2nd edition, in: *Kindai Nihon gakajutsu yōgo shusei* 近代日本學術用語集成 (Compendium of modern Japanese scientific terminologies). Tokyo: Ryūkai shosha. (b) Cf. Nihon sugakai 日本數學會 (ed.). *Iwanami sūgaku jiten* 岩波數學辭典 (Encyclopaedic dictionary of mathematics). 2nd edition. Tokyo: Iwanami shoten. (c) Cf. the 'magic square' in ancient Chinese mathematics *zongheng tu* 縱横圖 . (d) Same translation as in the *Jihe yuanben* 幾何原本 (1607), a translation of the first six books of Euclid's *Elements* by Xu Guangqi and Matteo Ricci. (e) In the translation of the *Elements*, a 'proposition' was called *ti* 題 , a 'lemma' was translated by *zeng ti* 增題 .

Table 3: Terminology for 'quadratic' series

Seki Takakazu (1683)[a]	*Zhu Shijie (1303)*[b]	*Li Shanlan (1867)*
圭垜 *keida*	茭草 *jiaocao* 1+2+3+4+...+n	元垜 *yuanduo*
平方垜 *heihōda*	四角 *sijiao* 12+22+32+42+...n2	一乘方垜 *yicheng fangduo*
立方垜 *rippōda*	13+23+33+43+...n3	二乘方垜 *ercheng fangduo*
三乘方垜 *sanjō hōda*	14+24+34+44+...n4	三乘方垜 *sancheng fangduo*
四乘方垜 *yonjō hōda*	15+25+35+45+...n5	四乘方垜 *sicheng fangduo*
...	...	...

Notes: (a) Expressions are taken from the chapter "Hōda jutsu" 方垜術 (Procedures for n-th power-piles) in Seki 1974, *juan* 1. (b) Only the two upper series (*jiaocao* and *sijiao*) appear in Zhu's *Jade Mirror*. The others are later extensions.

Table 4: Terminology for 'triangular' series

Seki Takakazu (1683)[a]	*Zhu Shijie (1303)*	*Li Shanlan (1867)*
圭垜 *keida*	茭草 *jiaocao* 1+2+3+4+ …+n	一乘垜 *yichengduo*
三角衰垜 *sankaku suida*	三角 *sanjiao* 1+3+6+10+…+ (1+2+…+n)	二乘垜 *erchengduo*
再乘衰垜 *saijō suida*	撒星 *saxing* (1+4+10+…+[1+(1+2)+ …+(1+…+n)]	三乘垜 *sanchengduo*
三乘衰垜 *sanjō suida*	三角撒星 *sanjiao saxing* 1+5+15+…+[1+(1+(1+2))+ …+(1+…+(1+…n))]	四乘垜 *sichengduo*
四乘衰垜 *yonjō suida*	三角撒星更落一 *sanjiao saxing gengluo yi* …	五乘垜 *wuchengduo*
…	…	…

Note: (a) Expressions taken from the chapter "Suida jutsu" 衰垜術 (Procedures for sequential piles) in Seki 1974, *juan* 1.